CAIIB PAPER-1

Module-A Unit-8

Advanced Bank Management (ABM)

CAIIB Paper 1 (ABM) Module A Unit 8: Linear Programming

Introduction

Linear Programming refers to several related mathematical techniques that are used to allocate limited resources among competing demands in an optimal way. For obtaining the optimal solution the problems should be structured into a particular format. It has been found that linear programming has many useful applications to financial decisions. The type of problems should have linear constraints and the decision maker must be trying to maximise some linear objective function.

In this chapter we will discuss graphical and '**simplex' methods**.

Model

Let us assume that the selling prices, production and marketing costs are known for each of the 'n' products. The firm also has to operate under certain economic, financial and physical constraints. Some examples of resource and marketing constraints:

- Bank may stipulate certain working capital requirements.
- Market may not absorb the whole output.
- Capacity constraints.
- Labour availability.
- Raw materials availability.

These constraints can be used to formulate the problem. The question is how to attain maximum profit minimum loss or minimum cost or time in the given circumstances? Maximum or minimum value can be obtained by forming and solving Linear Programming Problem.

Thus, Linear Programming Problem is a method by which a function (profit, loss, time, cost, etc.) can be maximised or minimised (optimised) with respect to some conditions. The function which has to be maximised or minimised (optimised)) is called objective function and the conditions are called constraints. The variables related to a linear programming problem whose values are to be determined are called Decision variables.

Under what conditions a Linear Programming problem can be formulated?

- As the name implies all equations are linear – This implies proportionality. For example, if it takes 4 persons to produce one unit, then we require 12 persons to produce 3 units.
- The constraints are known and deterministic. That is, the probabilities of occurrence are presumed to be 1.0.
- Most important rule is that all these variables should have non-negative values.
- Finally, decision variables are also divisible.

Graphic Approach

Let us illustrate the graphic approach with simple numerical two-decision variables. (3 variables require 3-D graphing). This gives a quick insight into the nature of L.P.

Let firm A produce radios and television sets.

Each radio costs Rs. 500 in wages and Rs. 500 in materials.

Each television set costs Rs. 2,500 in wages and Rs. 1,500 in materials.

The firm pays the labour and material expenses in cash.

The price of a radio is Rs. 2,000 and the price of a television is Rs. 6,000.

As there is a strong consumer demand, the firm is able to sell as many units as it produces at prevailing prices.

The firm also gives one period credit to consumers. The firm has 10 hours of machine time and 4 hours of assembly time per day.

The production of radio requires 3 hours of machine time and 1 hour of assembly time. The production of television requires 1 hour of machine time and 1 hour of assembly time.

The firm has Rs. 12,000 as cash balance (liquidity to pay for labour and materials). Now, given the financial and capacity constraints, how many radios and televisions should the firm produce in period 1, to maximise its profits?

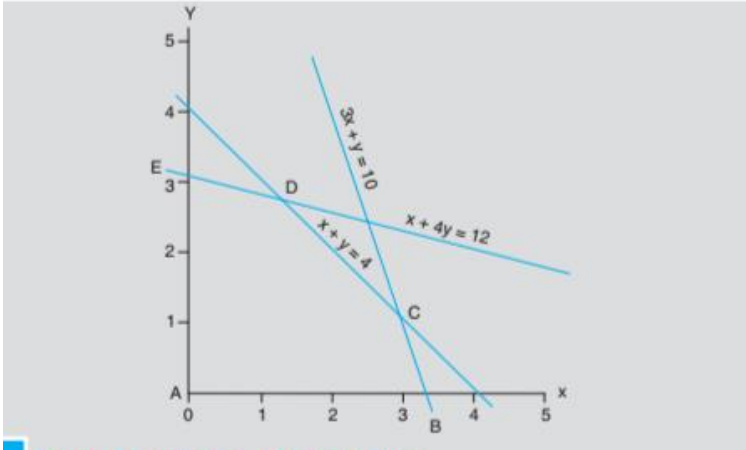
Let x and y be respectively, the units of radios and television sets produced in period 1. Then the constraints are:

(a) (capacity constraint machine time) 3x + y £ 10

(b) (capacity constraint assembly time) x + y £ 4

(c) (financial constraint) 1000x + 4000y £ 12,000 ® same as x + 4y £ 12

(d) (non-negativity) x ^{3} 0; y ^{3} 0;



1 Graphic Determination of Optimal Solution

(e) Objective function: Maximise Profit = 1,000x + 2,000y Now, let us draw the graph.

Line 1	$x + y = 4$				
Data	x	0	2	4	
	y	4	2	0	
Line 2	$3x + y = 10$				
Data	x	3	2	1	0
	y	1	4	7	10
Line 3	$x + 4y = 12$				
Data	x	0	4	8	12
	y	3	2	1	0

We have plotted the above three constraints in the graph. Find all the combinations of x and y, which satisfy the constraint and plot the points for all 3 lines. The graph is in the 1st quadrant. This satisfies the non-negativity condition.

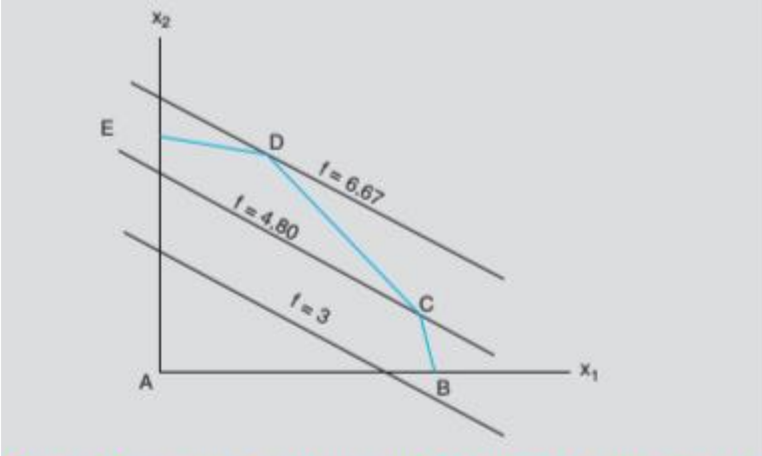
- All points on or below (inside) the line satisfy, x + y £ 4.
- All points on or below the line 3x + y £ 10, satisfy the machine time constraint.
- All points on or below the line x + 4y £ 12, satisfy financial constraint.

Even though all constraints are listed separately, they should be satisfied simultaneously. When these restrictions are placed one on top of the other, we obtain a common area, which in this case is shaped like a pentagon. (say ABCDE). Every point in this pentagon satisfies the constraints. This area is referred to as a set of feasible solutions.

Now, our objective is not to pick any feasible solution.

Although x = y = 0 is also a feasible solution, the profit will be zero.

This means no production of either radio or television. We are not seeking such a solution. So, our objective is to pick that feasible solution (that particular combination of x and y), from the set of feasible solutions, which maximises profit.



Graph of Constraints, Non-negativity Conditions and ISO Profit Line

Simplex Method

Another method of solving linear programming is Simplex Method. This method is a standard technique in linear programming for solving an optimisation (maximisation or minimisation) problem, typically one involving an objective function and several constraints expressed as inequalities. With computer programmes, spread sheets available, it is possible to use this method effectively and solve equations with as many as 10–12 variables.

Let us take the following problem to use Simplex Method.

Problem

A company manufactures cricket bats and chess sets. Each cricket bat gives a profit of Rs. 2 and chess set gives a profit of Rs. 4.

	Workshop 1 (hr)	Workshop 2 (hr)	Workshop 3 (hr)
Availability (Per day)	120	72	10
Cricket Bat	4	2	0
Chess Set	6	6	1

If the company wants to maximise the profit, how many cricket bats and chess sets should be produced per day?

Step 1 Solution: Formulate the problem.

Let the production be 'B' bats and 'C' chess sets.

(a) Objective function: Maximise Z = 2B + 4C

(b) 4B + 6C £ 120 (Workshop 1)

(c) 2B + 6C £ 72 (Workshop 2)

(d) 1C £ 10

(e) B, C ^{3} 0

We now change this to standard LP format.

In the standard LP form, all the constraints are converted into equations with the help of slack variables. Also make sure that these equations have non-negative right hand side. For example, 4B + 6C £ 120 is changed to 4B + 6C + m = 120 Here m is called a slack variable. It takes non-negative values. In fact all the variables in these equations take non-negative values.

The standard LP format is as follows:

(a) Objective function Maximise Z = 2B + 4C + 0 m + 0 n + 0 p

(b) 4B + 6C + 1m = 120 (Workshop 1)

(c) 2B + 6C +1n = 72 (Workshop 2)

(d) 1C + 1p = 10

(e) B, C ^{3} 0; m, n, p ^{3} 0 where m, n, p are the slack variables.

Z equation is also written as Z – 2B – 4C – 0 m – 0 n – 0 p = 0. Now, make a tableau as follows

Basic variables	Z	B	C	m	n	p	Solution
Z	1	–2	–4	0	0	0	0
m	0	4	6	1	0	0	120
n	0	2	6	0	1	0	72
p	0	0	1	0	0	1	10

This tableau gives the coefficients of the variables *Z*, *B*, *C*, *m*, *n*, *p* in the four equations written in the standard LP format, starting with the *Z*-equation. This tableau is a convenient way of setting up the information. This gives,

1. The variables which are in the solution at that point. (*Z*, *m*, *n*, *p*)
2. Profit associated with the solution. (0 when $B = 0$, $C = 0$)
3. The variable that will add most to profit, if brought into the solution. This is indicated by the variable which has most negative coefficient in the *Z*-row.

Here the most negative coefficient is –4 for *C*. So *C* is called the entering variable. Next, we need to rewrite the tableau by replacing one of the basic slack variables by *C*.

To decide which current basic variable is to be replaced by *C*, we concentrate on the *C*-column and the solutions column. Take the ratio of the corresponding entries in w these columns. Look at the following table:

C	Solution	Ratio
–4	0	
6	130	120/6 = 20
6	72	72/6 =12
1	10	10/1 = 10

Then we choose the smallest positive value in the ratio column, which is 10. The slack variable corresponding to this is *p*.

Thus we decide to replace *p* by *C*. Look at the tableau below which is a reproduction of the previous one. We have highlighted the column under C, and the p-row, which is called the pivot row. The intersection of the highlighted row and column is called the pivot entry, which is 1 here.

Basic variables	Z	B	C	m	n	P	Solution
Z	1	–2	–4	0	0	0	0
m	0	4	6	1	0	0	120
n	0	2	6	0	1	0	72
p	0	0	1	0	0	1	10

Now we form a new tableau where
(i) new pivot row = (current pivot row)/(pivot entry)
(ii) all other rows = current row – (pivot entry)*(new pivot entry)

Basic variables	Z	B	C	m	n	P	Solution
Z	1	–2	0	0	0	4	40
m	0	4	0	1	0	–6	60
n	0	2	0	0	1	–6	12
p	0	0	1	0	0	1	10

So we have completed one iteration of the problem.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