



Module-C Unit-3

JAIIB PAPER-3

Accounting & Financial Management for Bankers(AFM)



JAIIB Paper 3 (AFM) Module C Unit 3- Financial Mathematics — Calculation of Interest And Annuities

Meaning of Interest

- Interest can be defined as the price paid by a borrower for the use of a lender's money.
- It is compensation paid to the depositor.
- In another words, it is excess of money paid or received on deposits or borrowings.
- Interest is the price paid by a borrower for the use of a lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending).

Types of Interest

Interest can be of two types:

- Simple Interest
- Compound Interest

Simple Interest:

SI is interest earned on only the original amount, called Principal, lent over a period of time at a certain rate.

Formula = $PRT / 100$

P = Principal

Rate = rate

T = Time

Illustration:

Q.A student purchases a computer by obtaining a loan on simple interest.

The computer costs ₹ 60,000 and the interest rate on the loan is 12% per annum (simple). If, the loan is to be paid back after two years, calculate:

- The amount of total interest to be paid,
- The total amount to be paid back after 2 years,
- **Principal: 'P' = Rs. 60,000,**
- **Interest rate: 'R' = 12% = 0.12,**
- **Repayment time: T = 2 years**

Interest =

$$\text{PRT} = 60,000 \times 0.12 \times 2 = \text{Rs. } 14,400$$

Find the total amount to be paid back.

$$\text{Total repayments} = \text{Principal} + 60000 + 14400 = \text{Rs. } 74,400$$

Simple interest questions can be solved by applying the following formulas:

$$I = P \times rt$$

$$A = P + I \text{ Or,}$$

$$A = P + Prt \text{ Or,}$$

$$A = P(1 + rt)$$

$$I = A - P$$

A: Accumulated amount (final value of a deposit) at the end of the period

P: Principal (initial amount of the deposit)

r: annual interest rate expressed as a decimal fraction

I: interest after t years

Illustration 2:

Mohan invested Rs. 5,000 in a Company's fixed Deposit with an interest rate of 9.8%. How much interest would he earn in 2 years?

$$P = \text{Rs. } 5,000,$$

$$r = 9.8\%$$

$$t = 2 \text{ years}$$

Answer

$$I = P \times rt$$

$$I = (\text{Rs. } 5,000) (9.8\%) (2) = \text{Rs. } 980$$

Hence, Mohan would earn Rs. 980 in 2 years.

Compound Interest:

CI is interest earned on any previous interests earned as well as on the Principal lent. It is Interest on interest.

$$CI = P(1+i)^n - P$$

$$A = P(1+i)^n$$

P = Principal (Initial amount you borrowed or deposited)

r = Annual rate of interest (expressed as fraction, e.g. 6 per cent per annum = $6/100 = 0.06$)

n = Number of years for which the amount is deposited

A = Amount of money accumulated after n years including interest.

Frequently compounding of Interest

What if the interest is paid more frequently?

The accumulated amount (A), at the end of one year will be:

- Annually = $P(1 + r)$ = Annual compounding**
- Quarterly = $P(1 + r/4)^4$ = Quarterly compounding**
- Monthly = $P(1 + r/12)^{12}$ = Monthly compounding**
- Interest is computed on an account balance (i.e. a saving account).
- However, when interest is added to the principal amount lying in the account versus returning it immediately to the customer, the interest itself will earn interest till the next date for computing the interest. This is known as the compounding of the interest or more simply stated as compound interest.
- **Compounding Interest:** The time interval between which the interest is added to the account

The interest rate together with compounding period and balance in the account determines how much interest is added in each compounding period.

- The basic formula is $A = P(1 + r/n)^{nt}$
- P = the principal
- A = the amount accumulated (Principal + Interest)
- r = the rate (expressed as fraction, e.g. 6 per cent = 0.06)
- n = number of times per year the interest is compounded
- t = number of years for which the principal amount is invested
- Note: The above explanation is more easily understandable by thinking in terms of a simplified compound interest. When the interest is only compounded once per year ($n = 1$)

illustration simplifies.

- **$A = P(1 + r)t$ $A = 10,000(1 + 0.06)^1 = 10,600$**
- The following table shows the final accumulated amount (A) after $t = 1$ year of an account initially with $P = 10,000$ at 6 per cent interest rate with given compounding (n)

	n	A
1	(yearly)	10,600
2	(semiannually)	10,609
4	(quarterly)	10,613
12	(monthly)	10,616
52	(weekly)	10,618

- **If the applicable interest rate and the length of the deposit all remain the same, more interest is earned by increasing the number of compounding periods per year.**
- **Special Note:** When interest is compounded continually (in other words, when n approaches infinity) the compound interest equation takes the form $A = Pert$ where e is approximately 2.71828 (the exponential number).
- **The Rule of 72:** Allows you to determine the approximate number of years before your money doubles with yearly compounding. Here is how to do it; Divide the number 72 by the percentage rate you are paying on your debt (or earning on your investment).

Illustration:

You borrowed Rs.1,000 at 6% interest. Then, 72 divided by 6 is 12. That makes 12 the approximate number of years it would take for your debt to double to Rs. 2,000, if you did not make any payment.

Similarly, a saving account with Rs. 500 deposited in it, earning 4% interest and compounded yearly, will take 18 years for Rs. 500 to double to Rs. 1,000 if you do not make any further deposit, as 72 divided by 4 is 18.

Illustration

The simple interest in 3 years and the compound interest in 2 years on a certain sum at the same rate are Rs. 1,200 and Rs. 832 respectively. Find

- The rate of interest,
- The principal,
- The difference between the C.I. and S.I. for 3 years

$$(i) CI(2 \text{ year}) - SI(2 \text{ year}) = SI(2 \text{ year}) * r / 200 = PR^2 / 100^2$$

$$SI \text{ for 3 years} = 1200$$

$$\text{so for 2 years} = 800$$

$$832 - 800 = 800 * r / 200$$

$$32 = 4 * r$$

$$r = 8\%$$

$$(ii) CI(2 \text{ year}) - SI(2 \text{ year}) = PR^2 / 100^2$$

$$32 = P * 8^2 / 10000$$

$$= 5000$$

$$(iii) \text{ Amount due after 3 years} = Rs. 5,000 \times (1 + R)^3 =$$

$$= 5,000 \times 1.2597$$

$$= Rs. 6,298.56$$

$$\text{Hence, C.I. for 3 years} = A - P = 6,298.56 - 5,000 = Rs. 1,298.56$$

$$\text{The difference between the C.I. and S.I. for 3 years} = 1,298.56 - 1,200 = Rs. 98.56$$

Illustration

The population of an industrial town is increasing by 5% every year. If the present population is 1 million, estimate the population five years hence. Also, estimate the population three years ago.

Solution

Present population, P = 1 million, rate of increase = 5% per annum Hence, the population after 5 years = $10,00,000 (1.05)^5 = 12,76,280$

Population three years ago = $10,00,000 / (1.05)^3 = 8,63,838$

Since the population three years ago, compounded at 5 per cent, is equal to 1 million, today.

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Fixed And Floating Interest Rates
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There are two different modes of interest.

- Fixed Rates
- Floating Rates also called as variable rates.

Fixed Rate:

- In the fixed rate, the rate of interest is fixed. It will not change during the entire period of the loan.
- For example, if a home loan, taken at an interest rate of 12 per cent, is repayable in 10 years, the rate will remain the same for the entire tenure of 10 years even if the market rate increases or decreases. The fixed rate is, normally, higher than the floating rate, as it is not affected by market fluctuations.

Floating Rate:

- In the floating rate or variable rate, the rate of interest changes, depending upon the market conditions. Under floating rate, the interest rate is usually linked to a

benchmark rate which could be the MCLR/base rate of the bank or any other benchmark rate of the banking industry.

- It may increase or decrease depending upon the change in the benchmark rate.
- For example, if a home loan is taken at an interest rate of 12 per cent, repayable in 10 years, in April 2021, and if the benchmark rate increases to 12.5 per cent in April, 2022, the interest rate of this loan will also be increased to 12.5 per cent. If the loan is under an EMI system, depending upon the change in interest rate, the repayment period varies, but equated monthly instalment remains the same. However, the borrower may choose to have the repayment period same and pay a higher EMI.

Front-End And Back-End Interest Rates

If the interest is deducted from the principal amount and only the net amount is disbursed, it is **called front-end interest**.

For example, when the bank discounts a bill, the interest applicable for the tenure of the bill is calculated and is deducted from the bill amount along with other charges and the net amount is paid to the customer.

However, the normal practice in banking industry is to charge back-end interest rate which means that the full amount of the loan is disbursed and the interest is charged subsequently on monthly/quarterly/agreed basis.

For example, in a term loan, the interest is calculated on the actual daily balances in the account during a period and applied at the end of the period. Obviously, the front-end interest application results in effective interest rate being more as the borrower gets less amount for use whereas, the interest is applied on the full amount.

Calculation Of Interest Using Products/Balances

- Calculation of front-end interest like in bill discounting is easy as the amount is assumed to be constant over the entire period.
- For example, if the tenure of the bill of Rs.2 lac is 3 months and the rate of discount is 16% p.a., the interest amount will be Rs.8,000.
- In banks, many of the cases of deposit and loan accounts involve calculation of interest on the basis of daily balance in the customer's account.
- While this method was prevalent in case of the loan accounts, even in case of Savings Account, the interest is now required to be calculated on the basis of daily balances.
- In this method, the closing balance in the account is multiplied by the number of days for which that balance remains unchanged.
- Such products are calculated for the entire period for which the interest is to be applied.

- The sum of such products is multiplied by the applicable interest rate and divided by 365(No. of days in a year).

The following illustration will clarify this. Illustration The following table shows date-wise closing debit balances in the cash credit account of customer for the month of June 2022, and the calculation of products.

Date	Balance	No. of Days	Product
1/6/2019	10,000	5	50,000
6/6/2019	15,000	4	60,000
10/6/2019	12,000	5	60,000
15/6/2019	10,000	7	70,000
22/6/2019	22,000	4	88,000
26/6/2019	20,000	4	80,000
30/6/2019	18,000	1	18,000

The total of the products is 426,000.

If the interest rate is 15% p.a., the interest for the month of June 2022 will be $(426,000 * 0.15) / 365 = \text{Rs. } 175.1$.

Alternatively,

$$[(10000 * 15\% * (5/365)) + (15000 * 15\% * (4/365)) + \dots + (18000 * 15\% * (1/365)) = 175.1$$

Annuities

What are Annuities?

At some point in your life, you may have had to make a series of fixed payments over a period of time –such as rent or car payments-or have received a series of payments over a period of time, such as bond coupons. These are called annuities. (Regular periodic payment)

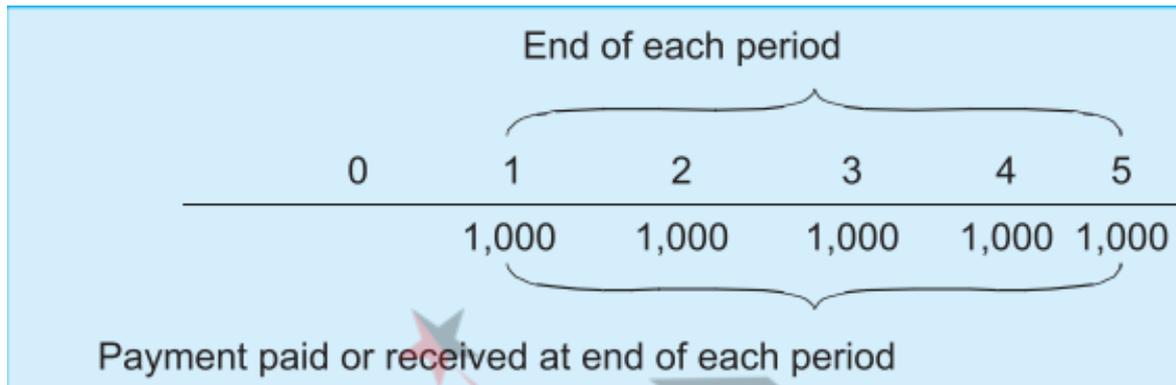
- **Ordinary Annuity:** Payment are required at the end of each period. For an illustration, straight bonds usually make coupon payments at the end of every six months until the bond's maturity date.
- **Annuity Due:** Payments are required at the beginning of each period. Rent is an illustration of annuity due. You are usually required to pay rent when you first move in at the beginning of the month, and then on the first of each month thereafter.

Calculating The Future Value Of An Ordinary Annuity

- If you know how much you can invest per period for a certain time period, the future value of an ordinary annuity formula is useful for finding out how much you would have in the future by investing at your given interest rate.
- If you are making payments on a loan, the future value is useful for determining the total cost of the loan.

Let us now run through the illustration 1.

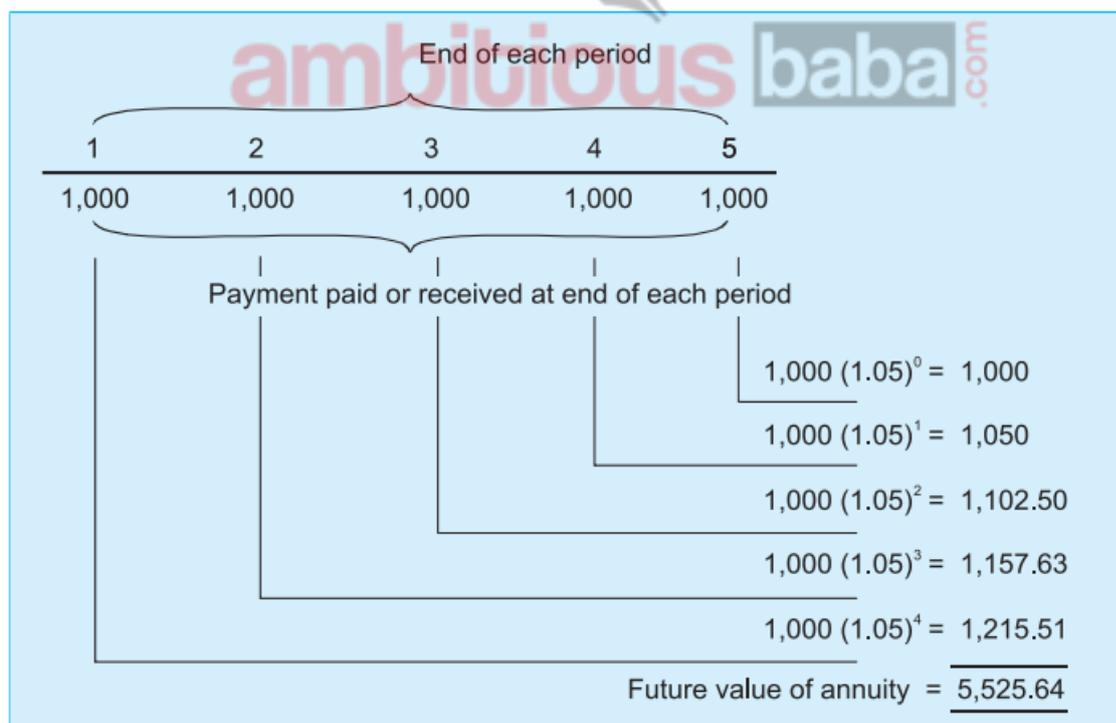
Consider the following annuity cash flow schedule:



In order to calculate the future value of the annuity, we have to calculate the future value of each cash flow.

Let us assume that you are receiving Rs. 1,000 every year for the next five years, and you invest each payment at 5%.

The following diagram shows how much you would have at the end of the five-year period:



$$FV (\text{Ordinary Annuity}) = C * \left[\frac{(1+i)^n - 1}{i} \right]$$

C = Cash flow per period

i = interest rate

n = number of payments

If we were to use the above formula for the illustration 1 above, this is the result:

$$\begin{aligned} FV (\text{Ordinary Annuity}) &= 1,000 * \left[\frac{(1+.05)^5 - 1}{.05} \right] \\ &= 1,000 * [5.53] \\ &= 5,525.63 \end{aligned}$$

Q.If Rs. 10000 is deposited at the end every year, how much amount will be received after 5 years at 10% p.a. interest.

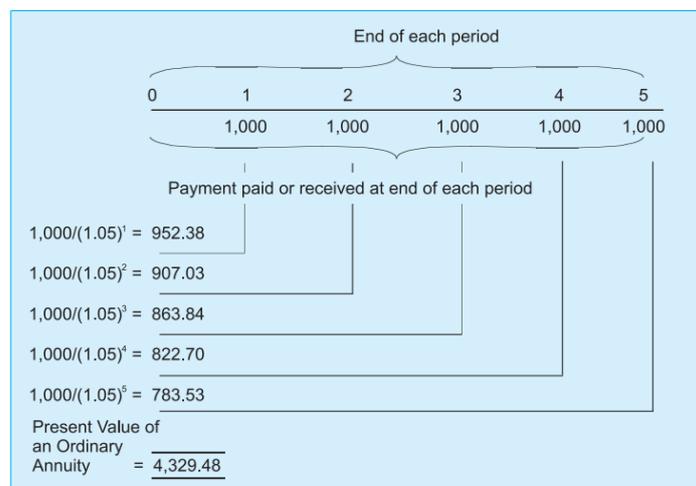
$$= 10000 [1.10^5 - 1] / .10$$

$$= \text{Rs.}61051$$

Calculating The Present Value Of An Ordinary Annuity

- If you would like to determine today's value of a series of future payments, you need to use the formula that calculates the present value of an ordinary annuity.
- You would use this formula as part of a bond pricing calculation.
- The PV of ordinary annuity calculates the present value of the coupon payments that you will receive in the future.

For the illustration



$$PV (\text{Ordinary Annuity}) = C * \left[\frac{(1+r)^n - 1}{r(1+r)^n} \right]$$

C = Cash flow per period

r = interest rate

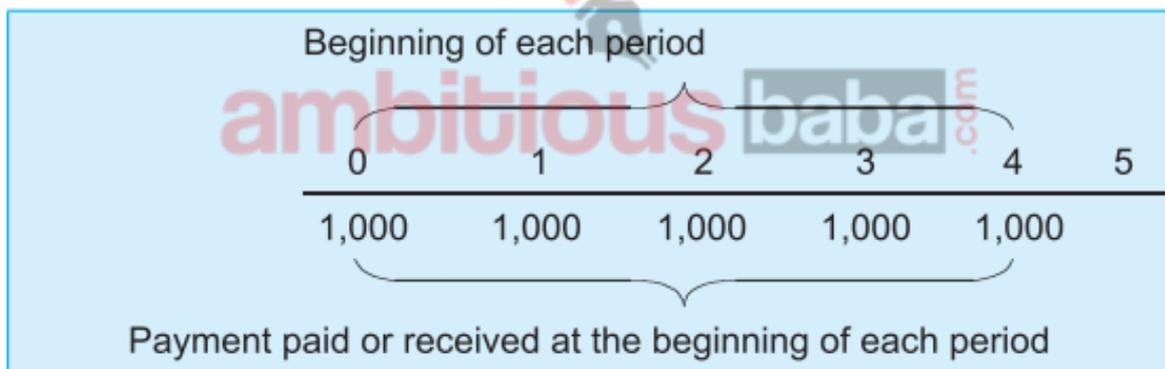
n = number of payments

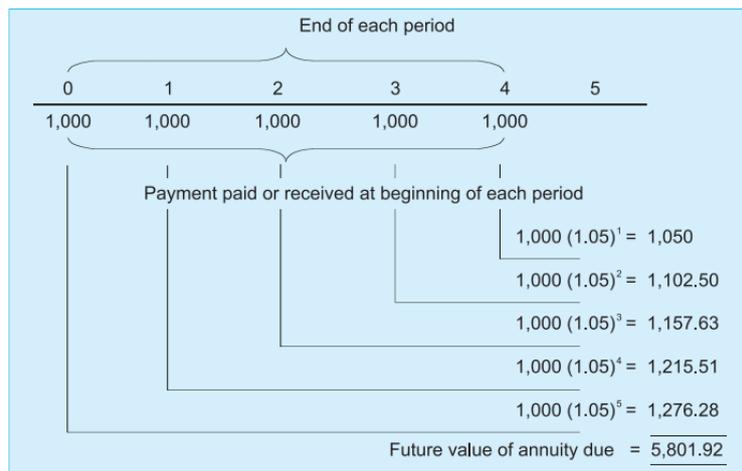
The formula provides us with the PV in a few easy steps. Here is the calculation of the annuity represented in the diagram for Illustration 2:

$$\begin{aligned} PV (\text{Ordinary Annuity}) &= 1,000 * \left[\frac{(1+.05)^5 - 1}{0.05(1.05)^5} \right] \\ &= 1,000 * \left[\frac{0.27628}{0.05(1.27628)} \right] \\ &= 1,000 * \frac{0.27628}{0.063814} \\ &= 1,000 * 4.3295 \\ &= 4,329.45 \end{aligned}$$

Calculating The Future Value Of An Annuity Due

- When you are receiving or paying cash flows for an annuity due, your cash flow schedule would appear as follows: Since, each payment in the series is made one period sooner; we need to discount the formula one period later.
- A slight modification to the FV-of-an-ordinary-annuity formula accounts for payments occurring at the beginning of each period.





$$FV (\text{Annuity Due}) = C * \left[\frac{(1+i)^n - 1}{i} \right] * (1+i)$$

Therefore,

$$\begin{aligned}
 FV (\text{Annuity Due}) &= 1,000 * \left[\frac{(1+.05)^5 - 1}{.05} \right] * (1+.05) \\
 &= \text{Rs. } 1,000 * 5.53 * 1.05 \\
 &= \text{Rs. } 5,801.91
 \end{aligned}$$

Calculating The Present Value Of An Annuity Due

- For the present value of an annuity due, we need to discount the formula one period forward, as the payments are held for a lesser amount of time.
- When calculating the present value, we assume that the first payment made was today.
- We could use this formula for calculating the present value of your future rent payments as specified in a lease you sign with your landlord. Let us say for the illustration that you make your first rent payment at the beginning of the month and are evaluating the present value of your five-month lease on that same day.
Your present value calculation would work as follows:

End of each period				
0	1	2	3	4
1,000	1,000	1,000	1,000	1,000
Payment paid or received at the beginning of each period				
$1,000/(1.05)^0 = 1,000$				
$1,000/(1.05)^1 = 952.38$				
$1,000/(1.05)^2 = 907.03$				
$1,000/(1.05)^3 = 863.84$				
$1,000/(1.05)^4 = 822.70$				
Present Value of an annuity due				
=	<u>4,545.95</u>			

$$PV (\text{Annuity Due}) = C * \left[\frac{(1+r)^n - 1}{r(1+r)^n} \right] \times (1+r)$$

Therefore,

$$\begin{aligned}
 PV (\text{Annuity Due}) &= 1,000 \times \left[\frac{(1.05)^5 - 1}{.05(1.05)^5} \right] \times 1.05 \\
 &= 1,000 \left[\frac{0.27628}{0.063814} \right] \times 1.05 \\
 &= 1,000 \times 4.3295 \times 1.05 = 4,545.92
 \end{aligned}$$

$$FV (\text{Ordinary Annuity}) = C * \left[\frac{(1+i)^n - 1}{i} \right]$$

$$PV (\text{Ordinary Annuity}) = C * \left[\frac{(1+r)^n - 1}{r(1+r)^n} \right]$$

$$FV (\text{Annuity Due}) = C * \left[\frac{(1+i)^n - 1}{i} \right] * (1+i)$$

$$PV (\text{Annuity Due}) = C * \left[\frac{(1+r)^n - 1}{r(1+r)^n} \right] \times (1+r)$$

Repayment Of A Debt

A debt is required to be repaid as per the terms of the contract with lender.

In the banking industry in India, the following three methods of repayment are common.

- ✓ Equal monthly/quarterly installment of principal plus the interest applied during the period.
- ✓ Equated monthly/quarterly installment covering both the principal and the interest.
- ✓ Bullet/balloon repayment under which the entire loan amount is repaid at the end of the period with accumulated interest. Alternatively, the interest is paid periodically, as and when applied, and the principal amount of the loan is paid at the end of the contract period.

These are discussed below in detail.

Equal monthly/quarterly installment of principal plus the interest applied during the period.

Illustration

Your friend has borrowed Rs.1,000 from you and wants to repay you on a monthly basis rather than the whole amount all at once at the end of the year. The important point here is that he will owe you less in principal each month.

The applicable rate of interest 8% p.a. means 0.667% per month.

Calculation:

The principal payment each month will be Rs. 83.33 (1,000/12)

First month: Interest = $1,000 \times 0.667\% \times 1 = 6.67 + 83.33 = \text{Rs. } 90$.

The principal owed at the end of the month is Rs. 916.67.

Second month: Interest = $916.67 \times 0.667\% \times 1 = 6.11 + 83.33 = 89.44$.

Third month: Interest = $833.34 \times 0.667\% \times 1 = 5.56 + 83.33 = 88.89$.

And so on for the twelve months.

Balance	Principal Payment	Interest Payment	Total Payment
1,000.00	83.33	6.67	90.00
916.67	83.33	6.11	89.44
833.34	83.33	5.56	88.89
750.01	83.33	5.00	88.33
666.68	83.33	4.44	87.77
583.35	83.33	3.89	87.22
500.02	83.33	3.33	86.66
416.69	83.33	2.78	86.11
333.36	83.33	2.22	85.55
250.03	83.33	1.67	85.00
166.70	83.33	1.11	84.44
83.37	83.33	0.56	83.89
	999.96	43.34	1,043.30

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